

On Predicting the Solar Cycle using Mean-Field Models

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ABSTRACT

We discuss the difficulties of predicting the solar cycle using mean-field models. Here we argue that these difficulties arise owing to the significant modulation of the solar activity cycle, and that this modulation arises owing to either stochastic or deterministic processes. We analyse the implications for predictability in both of these situations by considering two separate solar dynamo models. The first model represents a stochastically-perturbed flux transport dynamo. Here even very weak stochastic perturbations can give rise to significant modulation in the activity cycle. This modulation leads to a loss of predictability. In the second model, we neglect stochastic effects and assume that generation of magnetic field in the Sun can be described by a fully deterministic nonlinear mean-field model — this is a best case scenario for prediction. We designate the output from this deterministic model (with parameters chosen to produce chaotically modulated cycles) as a target timeseries that subsequent deterministic mean-field models are required to predict. Long-term prediction is impossible even if a model that is correct in all details is utilised in the prediction. Furthermore, we show that even short-term prediction is impossible if there is a small discrepancy in the input parameters from the fiducial model. This is the case even if the predicting model has been tuned to reproduce the output of previous cycles. Given the inherent uncertainties in determining the transport coefficients and nonlinear responses for mean-field models, we argue that this makes predicting the solar cycle using the output from such models impossible.

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1. Introduction

Magnetic activity in the Sun is known to play a central role in driving both long-term and short-term dynamics (Tobias 2002; Weiss 2002). The magnetic field is responsible for spectacular events such as sunspots, solar flares, and coronal mass ejections, and for heating the solar corona to high temperatures. Large-scale magnetic activity is known to be dominated by the eleven year activity cycle. This cycle has been systematically observed since the early seventeenth century and its properties are well documented (see e.g. Ossendrijver 2003). Of particular current interest is the impact of magnetic activity on solar irradiance that might have significant implications for the terrestrial climate (see Solanki *et al* 2004).

Given the importance of solar activity, it is not surprising that there has been a continued interest in understanding the mechanisms responsible for generating the solar magnetic field. The Sun’s magnetic field is believed to be generated by a hydromagnetic dynamo in which motion of the solar plasma (advection) is able to sustain a magnetic field against the continued action of ohmic dissipation (see e.g Moffatt 1978; Charbonneau 2005). Progress in understanding this fundamental problem of solar magnetohydrodynamics is slow owing to the difficulties of the dynamo problem. The extreme parameters of the solar interior and the inherent three-dimensionality of the dynamo problem make it impossible to solve the equations accurately on a computer. Much effort has therefore focused on *mean-field* dynamo models (Steenbeck, Krause & Rädler 1966; Krause & Rädler 1980), which describe the evolution of the mean magnetic field, parameterising the effects of the small-scale fields and flows in terms of tensor transport coefficients. These transport coefficients include α_{ij} (which leads to a regenerative term in the mean-field equations — the so-called α -effect) and the turbulent diffusivity (β_{ijk}). We stress here that there is no mechanism within the theory for determining the form of these coefficients, except for flows at low magnetic Reynolds number or with short correlation time, and in solar models these are usually chosen in a plausible but ad-hoc manner (often, for simplicity, adopting isotropic representations in which $\alpha_{ij} = \alpha\delta_{ij}$ and $\beta_{ijk} = \beta\epsilon_{ijk}$). Much attention has been focused upon determining these transport coefficients in both the linear and nonlinear dynamo regimes from numerical simulations (Cattaneo & Hughes 1996; Brandenburg & Subramanian 2005) but there is still no consensus over the nature of these, even to within an order of magnitude (see Courvoisier, Hughes & Tobias 2006). Mean-field models have, however, proved successful in providing illustrations of the type of behaviour that might be expected to occur in the Sun (and other stars). It is often argued that, *although these models have no predictive power*, understanding the underlying mathematical form of the equations can lead to the identification of robust patterns of behaviour.

Many different models have been proposed for the solar dynamo. In the distributed

dynamo model, the α -effect operates throughout the convection zone and interacts with the latitudinal shear (or the sub-surface shear layer, see Brandenburg 2005) to generate magnetic field. Alternatively, the dynamo could be operating near the tachocline, where an α -effect might be driven either by a tachocline-based instability or by turbulent convection. This, in conjunction with the strong shear, could drive an “interface” dynamo (Parker 1993). Finally, there are flux transport models, in which the (so-called) Babcock-Leighton mechanism produces an α -effect (or source term) at the surface. This surface α -effect is coupled to the radial shear in the tachocline (where another α -effect may be operating) via a meridional flow (Choudhuri, Schüssler & Dikpati 1995; Dikpati & Charbonneau 1999). The relative merits of these models are discussed elsewhere in the literature (see, e.g. Charbonneau 2005) — the only comment we make here is that this plethora of models arises because of the lack of available constraints on the form of the transport coefficients in the mean-field formalism. We note further that it is not clear that any of the above scenarios capture the essential dynamo processes correctly or that these processes can ever be captured by a mean-field model.

It is also possible to construct predictions of solar activity without using dynamo theory, and there is a long literature describing these predictive methods (see e.g. Zhang 1996; Hathaway, Wilson & Reichmann 1999; Sello 2003; Zhao *et al* 2004; Saba, Strong & Slater 2005). One class of prediction techniques uses statistical and timeseries analysis methods (see e.g. Tong 1995 for details). These methods, which are also applicable in many other areas of physics, vary in complexity from simple linear methods to methods that use dynamical systems theory to reconstruct nonlinear attractors in phase space. However, these methods have the drawback that they do not utilise any of the “physics” of the problem. Predictions can also be made by using precursor methods (see e.g. Schatten 2002), which do utilise some of the physical features of the system in addition to the timeseries data.

In recent papers (Dikpati, de Toma & Gilman 2006; Dikpati & Gilman 2006), an attempt has been made to unify these two approaches by utilising a mean-field model in order to make predictions about the future activity of the Sun. These papers describe an axisymmetric, mean-field model of a flux transport dynamo. Here the authors make use of the observations of magnetic flux at the solar surface to feed into a model of solar activity. The flux that is observed at the solar surface is advected by a parameterised meridional flow (which can be observed down to a certain depth) and interacts with a differential rotation profile that has been inferred from helioseismology. The magnetic flux also interacts with turbulence, the effects of which are parameterised by certain turbulent transport coefficients (representing the turbulent diffusivity and the α -effect). As with all current mean field models these turbulent transport effects have been parameterised in a plausible but ad-hoc manner, and are unconstrained by observations and indeed theory. The simplest predictive

scheme proposed by Dikpati *et al* (2006) therefore takes the form of a parameterised linear system forced by boundary observations. The implicit underlying philosophy here is that by reducing the correct physics for the generation of the solar activity cycle (i.e. a nonlinear self-excited dynamo) to such a scheme, predictions about future solar activity can be made.

In this paper we shall investigate the predictability of various dynamo models. We demonstrate that even when all the nonlinear physics of the solar dynamo is removed, problems remain for prediction owing to the increased importance of stochastic effects — even very weak stochastic perturbations can produce significant modulation in these linear-type models. We also discuss the best-case scenario for prediction where stochastic effects can be ignored, and demonstrate that in these cases prediction is still difficult owing to uncertainties in the input parameters of these parameterised mean-field models.

The paper is organised as follows. In the next section we describe (in a general way) the importance of modulation and the role of stochasticity and nonlinearity in solar dynamo models. In section 3 we investigate a flux transport model and demonstrate how the presence of even extremely weak noise can render predictions useless. In section 4 we consider the “best-case” scenario for prediction where noise does not play a role in the modulation — we demonstrate that more accurate prediction schemes may arise by using basic timeseries analysis techniques rather than from constructing mean-field models of the solar cycle. Finally, in section 5 we discuss the implications of our work for predictions of the solar cycle.

2. Problems for prediction and mechanisms for modulation

In this section, we discuss the problems that must be overcome by schemes designed to yield a prediction of future solar magnetic activity. Some of these problems arise owing to the nature of solar magnetic activity whilst others arise from the lack of a detailed theory that is capable of describing solar magnetic activity in such extreme conditions as those that exist in the solar interior.

It is clear that if the solar cycle were strictly periodic, with a constant amplitude, then it would be straightforward to predict future behaviour. However, all measurements of solar magnetic activity (both direct observations and evidence from proxy data) indicate that the variations in the magnetic activity do not follow a periodic pattern. Departures from periodicity may be driven either by perturbations or by modulation. For the case of a weakly perturbed periodic system, the dynamics is essentially captured by the periodic signal, with the small perturbations playing a secondary role. We distinguish this behaviour from a modulated signal in which there are significant departures from periodicity (often occurring

on longer timescales), with large variations in the observed amplitude of the signal. All the evidence from direct observations indicates that the solar cycle is strongly modulated. The amplitude of the solar cycle varies enormously over long timescales, an extreme example of this modulation was a period of severely reduced activity in the seventeenth century known as the Maunder Minimum. Proxy data from records of terrestrial isotopes, such as ^{10}Be and ^{14}C (see e.g. Beer 2000, Weiss & Tobias 2000, Wagner *et al* 2001), demonstrate that this modulation has been a characteristic feature of the solar magnetic activity over (at least) the last 20,000 years.

Mathematically there are only two possible sources for this strong modulation of the basic solar cycle (Tobias 2002). The modulation may arise either as a result of stochastic effects (see e.g. Ossendrijver & Hoyng 1996) or by deterministic processes (see e.g. Tobias, Weiss & Kirk 1995). In this context we define deterministic processes to be those that *are* captured by the differential equations of dynamo theory, with no random elements. Stochastic processes are those that occur on an unresolved length or timescale, and so can not be described by the differential equations without including a random element into the model.

It is well known that stochastic modulation can arise even if the deterministic physics that leads to the production of the basic cycle is essentially linear. This parameter regime is generally considered to be a good one for prediction, since any nonlinear effects are only playing a secondary role. However, in this stochastically-perturbed case, the small random fluctuations that lead to the modulation will have large short-term effects and render prediction extremely difficult, if not impossible. Conversely, if the modulation arises purely as a result of deterministic processes, then the underlying physics is nonlinear (or potentially non-autonomous) and this leads to difficulty in prediction owing to the possible presence of deterministic chaos and (more importantly) the difficulty of constructing accurate nonlinear models with large numbers of degrees of freedom.

In the next two sections we demonstrate the problems for prediction for dynamo models in both of the classes described above. In the next section we describe a flux transport model of the same type as the one used in the prediction scheme of Dikpati *et al* (2006) and we demonstrate that even very small random fluctuations can produce significant modulation, leading to extreme difficulties for prediction. We then, in section 4, go on to describe a model where the modulation arises owing to the presence of deterministic chaos and show that in this case, prediction using model fitting is a poor way to proceed, but some prediction is possible if it is possible to reconstruct the attractor for activity.

3. Prediction using a stochastically-perturbed flux transport dynamo model

3.1. The dynamo model

We assume initially that the modulated solar magnetic activity can be described by a stochastically-perturbed mean-field dynamo model. In this model, nonlinear effects are playing a secondary role, and all the modulation is being driven by the stochastic effects. The aim of this section is to assess whether or not models of this type can be used to make meaningful predictions of the solar magnetic activity. In these models, the evolution of the large-scale magnetic field is described by the standard mean-field equation (see, for example, Moffatt 1978),

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B} + \mathbf{U} \times \mathbf{B} - \beta \nabla \times \mathbf{B}). \quad (1)$$

Here, \mathbf{B} represents the large-scale magnetic field and \mathbf{U} corresponds to the mean velocity field, β is the (turbulent) magnetic diffusivity, and the $\alpha \mathbf{B}$ term corresponds to the mean-field α -effect. Using the well-known $\alpha\omega$ approximation, we solve this equation numerically in an axisymmetric spherical shell ($0.6R_\odot \leq r \leq R_\odot$ and $0 \leq \theta \leq \pi$). In solving Equation (1) we need to ensure that \mathbf{B} remains solenoidal (i.e. $\nabla \cdot \mathbf{B} = 0$). To achieve this, we decompose the magnetic field into its poloidal and toroidal components,

$$\mathbf{B} = B(r, \theta, t) \mathbf{e}_\phi + \nabla \times (A(r, \theta, t) \mathbf{e}_\phi), \quad (2)$$

where $B(r, \theta, t)$ denotes the toroidal (azimuthal) field component and the scalar potential $A(r, \theta, t)$ relates to the poloidal component of the magnetic field. So, rather than solving Equation (1) directly, the problem has been reduced to solving two coupled partial differential equations for the scalar quantities $A(r, \theta, t)$ and $B(r, \theta, t)$. We adopt idealised boundary conditions, in which $A = B = 0$ at $\theta = 0$ and $\theta = \pi$ and $r = 0.6R_\odot$ and A and B are smoothly matched to a potential field at $r = R_\odot$.

This particular dynamo model is closely related to the flux transport model described by Dikpati & Charbonneau (1999). The large-scale velocity field, \mathbf{U} , is given by

$$\mathbf{U} = u_r(r, \theta) \mathbf{e}_r + u_\theta(r, \theta) \mathbf{e}_\theta + \Omega(r, \theta) r \sin \theta \mathbf{e}_\phi, \quad (3)$$

where $\Omega(r, \theta)$ is a prescribed analytic fit to the helioseismologically-determined solar rotation profile (see, for example, Bushby 2006) and u_r and u_θ correspond to a prescribed meridional circulation. We assume that the meridional circulation pattern in each hemisphere consists

of a single cell, with a polewards flow at the surface and an (unobservable) equatorwards flow at the base of the convection zone — the flow is confined to the region $R_b \leq r \leq R_\odot$. The functional form that we adopt for this flow is similar in form to the one described by Dikpati & Charbonneau (1999),

$$u_r(r, \theta) = U_o \left(\frac{R_\odot}{r} \right)^2 \left[-\frac{2}{3} + \frac{1}{2}c_1\xi^{0.5} - \frac{4}{9}c_2\xi^{0.75} \right] \xi \sin \theta (3 \cos^2 \theta - \sin^2 \theta), \quad (4)$$

$$u_\theta(r, \theta) = U_o \left(\frac{R_\odot}{r} \right)^3 [-1 + c_1\xi^{0.5} - c_2\xi^{0.75}] \cos \theta \sin^2 \theta, \quad (5)$$

where $\xi(r) = [(R_\odot/r) - 1]$, $c_1 = 4[\xi(R_b)]^{-0.5}$, $c_2 = 3[\xi(R_b)]^{-0.75}$, and U_o is some characteristic flow speed. This flow pattern can be stochastically perturbed by setting $R_b = 0.7R_\odot + \epsilon(t)$, where $\epsilon(t)$ is a time-dependent, randomly fluctuating variable in the range $-0.005R_\odot \leq \epsilon \leq 0.005R_\odot$. The aim here is to assess whether or not such weak stochastic variations in the flow pattern could give rise to significant modulation in the activity cycle, and if so what are the consequences for prediction.

In order to complete the specification of the model, we need to choose plausible functional forms for the α -effect and the turbulent magnetic diffusivity. It should be emphasised again that these mean-field coefficients are poorly constrained by theory and observations, although plausible assumptions can be made. Defining β_o to be a characteristic value of the turbulent magnetic diffusivity within the solar convection zone, we adopt a similar spherically-symmetric profile to that adopted by Dikpati & Charbonneau (1999),

$$\beta(r) = \frac{1}{2}(\beta_o - \beta_c) \left[1 + \operatorname{erf} \left(\frac{r - 0.7R_\odot}{0.025R_\odot} \right) \right] + \beta_c, \quad (6)$$

where erf corresponds to the error function and β_c (here taken to be 1% of β_o) represents the magnetic diffusivity below the turbulent convection zone. Following Dikpati & Charbonneau (1999), rather than prescribing a simple functional form for α we neglect the α -effect term in the toroidal (B) field equation and replace the corresponding αB term in the poloidal (A) equation by a non-local, nonlinear source of poloidal flux,

$$S(r, \theta, t) = \frac{S_o}{2} \left[1 + \operatorname{erf} \left(\frac{r - 0.95R_\odot}{0.01R_\odot} \right) \right] \left[1 - \operatorname{erf} \left(\frac{r - R_\odot}{0.01R_\odot} \right) \right] \left[1 + \left(\frac{B(0.7R_\odot, \theta, t)}{B_o} \right)^2 \right]^{-1} \sin \theta \cos \theta B(0.7R_\odot, \theta, t). \quad (7)$$

Here, S_o is a characteristic value of this poloidal source and B_o represents the (somewhat arbitrarily chosen) field strength at which this non-local source becomes suppressed by the magnetic field. This source term parameterises the contribution to the poloidal magnetic flux due to the decay of active regions — the non-locality reflects the fact that active regions are believed to form as the result of buoyant magnetic flux rising from the base of the convection zone to the solar photosphere. See Dikpati & Charbonneau (1999) for a more detailed discussion of this source term, though again it must be stressed that the functional form and the nonlinear dependence are chosen in a plausible yet ad-hoc manner.

3.2. Numerical results

In order to carry out numerical simulations, we first non-dimensionalise this flux transport model. By using scalings similar to those described by Dikpati & Charbonneau (1999), it can be shown that the model solutions are fully determined by two non-dimensional parameters (once other parameters such as B_o have been selected). Denoting the equatorial angular velocity at the solar surface by Ω_{eq} , these non-dimensional parameters are the Dynamo number, $D = S_o \Omega_{eq} R_\odot^3 / \beta_o^2$, and the magnetic Reynolds number corresponding to the meridional flow, $Re = U_o R_\odot / \beta_o$. Here, we set $D = 7 \times 10^6$ and $Re = 5600$. In the absence of stochastic noise, this set of parameters produces a strong circulation-dominated dynamo in which the magnetic energy is a periodic function of time. Although the dynamo number is not weakly supercritical, nonlinear effects are not strong enough here to produce a modulated activity cycle — the primary role of the nonlinearity is to prevent the unstable dynamo mode from growing exponentially. We term such a model a “linear-type” model.

When weak stochastic effects are included in the model, the resulting activity cycle is indeed weakly modulated. This is illustrated in Figure 1, which shows the time-dependence of this solution. The time-series clearly illustrates that, although the amplitude of the “cycle minimum” only appears to be weakly time-dependent, there are significant variations in the peak amplitude of the magnetic energy time-series. These variations are qualitatively similar to those observed by Charbonneau & Dikpati (2000), who considered large amplitude random fluctuations in the flow pattern within the solar convection zone — the peak amplitude of these fluctuations was comparable with the peak amplitude of the flow. In this particular model, we have shown that even very weak stochastic variations in the centre of mass of the flow pattern can still produce significantly modulated behaviour. These stochastic effects are expected to become increasingly significant for dynamo numbers approaching critical. So, these models are obviously highly sensitive to the addition of stochastic noise.

In the absence of stochastic noise, the attractor (in phase space) for this solution is

two-dimensional, and the future behaviour of the solution at any instant in time is entirely determined by the current position of the system on the attractor. The same is not true when this system is perturbed by stochastic effects, and it clearly becomes much more difficult to predict the future behaviour of the system. Since the attractor of this stochastically perturbed solution cannot be unambiguously defined, another possible way of assessing the “predictability” of this solution is to look for a correlation between successive cycle maxima. Defining T_n to be the magnitude of the n^{th} cycle maximum, Figure 2 shows T_{n+1} as a function of T_n . It is clear from this scatter plot that there is no obvious correlation between the amplitudes of successive cycle maxima in this case. Since the modulation is being driven entirely by random stochastic forcing, this result is not surprising. This lack of correlation suggests that the behaviour of previous cycles cannot be used to infer the magnitude of the following one. This implies that even weak stochastic effects may seriously reduce the possibilities for solar cycle prediction in this linear-type regime.

4. Predictions using a deterministic dynamo model

4.1. The dynamo model

In the previous section, we demonstrated that even very weak stochastic perturbations to the meridional flow pattern can lead to a loss of predictability in a linear-type flux transport dynamo model. In that model, the modulation of the activity cycle was driven entirely by stochastic effects. As discussed in Section 2, the only other possible scenario is that the observed modulation is driven by nonlinear effects. This scenario, where the observed modulation is deterministic in origin, is the “best-case” scenario for prediction, as in this case the entirely unpredictable stochastic elements may be ignored. We stress again that, given that solar magnetic activity is significantly modulated, either deterministic or stochastic modulation must be considered in any realistic model (predictive or otherwise) of the solar cycle. So, in this section, we completely neglect stochastic effects and assume that the observed (chaotic) modulation in the solar magnetic activity can be described by a fully deterministic model in which any activity modulation (e.g. solar-like “Grand minima”) is driven entirely by nonlinear effects. The model that we use was described in detail in two recent papers (Bushby 2005, 2006), so we only present a brief description here. The exact details of the model are unimportant for our main conclusions.

Like the flux transport dynamo model from the previous section, this model describes an axisymmetric, mean-field, $\alpha\omega$ -dynamo in a spherical shell. Unlike the previous model, this model represents an “interface-like” dynamo that is operating primarily in the region around the base of the solar convection zone. It is worth mentioning again that (as discussed in the

introduction) there is still no general consensus regarding which of these dynamo scenarios is more likely to be an accurate representation of the solar dynamo. For this interface-like dynamo model, we neglect meridional motions, since they are poorly determined near the base of the solar convection zone. Like several earlier models (e.g. Tobias 1997; Moss & Brooke 2000; Covas *et al* 2000), this dynamo model includes the feedback (via the azimuthal component of the Lorentz force) of the mean magnetic field upon the differential rotation (Malkus & Proctor 1975). This nonlinear feedback is a crucial element of the model and, in the absence of stochastic effects, is the sole driver of modulation in the magnetic activity cycle. Denoting this magnetically-driven velocity perturbation by $V(r, \theta, t)$, the large-scale velocity field is given by

$$\mathbf{U} = [\Omega(r, \theta)r \sin \theta + V(r, \theta, t)] \mathbf{e}_\phi, \quad (8)$$

where (as in the previous model) $\Omega(r, \theta)$ represents an analytic fit to the solar differential rotation. Whilst the evolution of the large-scale magnetic field is again governed by Equation (1), an additional evolution equation is required for the velocity perturbation, V . This equation is given by

$$\begin{aligned} \frac{\partial V}{\partial t} = & \frac{1}{\mu_o \rho} [(\nabla \times \mathbf{B}) \times \mathbf{B}] \cdot \mathbf{e}_\phi + \frac{1}{r^3} \frac{\partial}{\partial r} \left[\nu r^4 \frac{\partial}{\partial r} \left(\frac{V}{r} \right) \right] \\ & + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left[\nu \sin^3 \theta \frac{\partial}{\partial \theta} \left(\frac{V}{\sin \theta} \right) \right], \end{aligned} \quad (9)$$

where ρ represents the fluid density (here taken to be constant), μ_o is the permeability of free space and ν represents the (turbulent) fluid viscosity.

In order to complete the model, the spatial dependence of the transport coefficients (α , β and ν) must also be specified. Again, we emphasise that there are no direct observational constraints relating to these coefficients — as noted in the introduction, there is no consensus as to their form and there is still a debate as to their order of magnitude (and even their sign). Having said that, it is possible to make some plausible assumptions for an “interface-like” dynamo model (see Bushby 2006 for more details). The precise choices of these parameters are unimportant for our main conclusions.

Having set up this model, it is possible to choose a set of parameters so that the solutions *do* reproduce some salient features of the solar dynamo (Bushby 2005, 2006). We stress here that, although the parameters have been chosen in a plausible manner, this dynamo model should not be regarded as an accurate representation of the solar interior

and is subject to many uncertainties. Furthermore we stress again that this is the case with *all* mean-field solar dynamo models. However we use this model as a useful tool to analyse the possibility of producing predictive models of the solar cycle. We proceed by choosing fiducial parameters and profiles for the turbulent transport coefficients that lead to “solar-type” magnetic activity, with chaotically modulated cycles and recurrent “Grand Minima”. We then integrate this model forward in time to produce a timeseries and designate this timeseries as the “target” run, which any subsequent model should be able to predict. This target run is shown in Figure 3, which shows a timeseries of the activity together with a reconstruction of the dynamo attractor in phase space. Although this solution is chaotically modulated, it is certainly no more chaotic than the equivalent attractor for the ^{10}Be data, which is a well-known proxy for solar magnetic activity (e.g. Beer 2000). Whilst the nonlinear effects are significant enough to drive the modulation, they are actually very difficult to detect. In this model, the cyclic component of the fluctuations in the differential rotation (which are driven by the nonlinear Lorentz force) are small compared with the mean differential rotation. This is consistent with observations of the (so called) torsional oscillations in the solar convection zone. Finally, note once more that, since the modulation is driven entirely by nonlinear effects, this model is specified exactly.

4.2. Numerical results

The question is then posed as to whether *any* mean-field model can be constructed that leads to meaningful predictions of the future behaviour of the target run. Clearly the best chance for a mean-field model being capable of predicting the future behaviour of the target run is to use *the exact model* that led to the target run data. Hence we test this model first, as all subsequent models will be inferior to this. We proceed by setting the model parameters to be those that generated the long test run, and consider the behaviour of solutions that are started from very similar points on the attractor. Some of the solutions are shown in Figure 4. This figure shows clearly that although the predictor solutions are able to track the target solution for a couple of activity cycles, the nature of the solutions means that the predictors and target solution diverge quickly after this time. This is not surprising behaviour. It is well-known that chaotic solutions have a sensitive dependence on initial conditions and that long-term prediction of such solutions is fraught with problems (see e.g. Tong 1995). What is clear is that simply using a model that is based upon mean-field theory will not work in the long term *even if the model is correct in every detail*. One might be able to predict one or two cycles ahead *if one has solved the problem of constructing an exact representation of the solar dynamo* but as noted above this is not an easy task.

We now turn to the related problem of short-term prediction. As discussed in the introduction there are large uncertainties in the form and amplitude of the input parameters for all mean-field models. What we investigate here is whether these uncertainties lead to significant difficulties in prediction even in the short term. Again we examine the best case scenario and consider a mean-field model for the predictor runs that has correctly parameterised the *form* of all the input variables (differential rotation, α -effect, turbulent diffusivity and nonlinear response). In addition these predictors have been given the correct input *values* for all-but-one of the parameters. Hence the predictor models are exactly the same as the target model with the exception of one input parameter that has been altered by 5%. This would be a staggeringly good representation should it be possible to achieve this for solar activity. Furthermore we increase the chances of the predictor being able to predict the future behaviour of the target solution by matching the two timeseries over a number of cycles. This is analogous to the procedure employed by Dikpati *et al* (2006) who cite support for their forecasting model by assuring that their model agrees with the solar cycle data for eight solar cycles — in reality this is not difficult to achieve with enough model parameters at one’s disposal. Figure 5 shows the results of integrating the predictor models for two different choices of incorrect parameter. Note that even though the predictor has been designed to reproduce the target over a number of cycles and that the predictor is very closely related to the target, there is still a good chance that it can get the next cycle incorrect, with significant errors in (particularly) the cycle amplitude. There are also clear variations in the cycle period, which obviously implies that the exact time between successive cycle maxima is also an unpredictable feature of the system.

We stress again that any mean-field model of solar activity includes transport coefficients that are still uncertain possibly to an order of magnitude (and certainly not to 5% accuracy). Although the incredible success of global and local helioseismology is placing restrictions on the form of the differential rotation and the meridional flows, it is unlikely in the foreseeable future that significant constraints will be put on the transport coefficients or their nonlinear response to the mean magnetic field.

5. Predictions using a reconstruction of the attractor

Having established that there are difficulties in obtaining reliable predictions by fitting mean-field models (even if the modulation is deterministic in origin), it is of interest to determine whether or not more reliable predictions could be obtained by utilising more general timeseries analysis techniques. In order to reconstruct an attractor from a given timeseries, it is necessary to define a corresponding phase space. There are various ways of

doing this, but given (any) discrete timeseries, $x(t)$, in which the data is sampled at intervals of Δt , the vector

$$\mathbf{X}(t) = [x(t), x(t - \Delta t), \dots, x(t - (d - 1)\Delta t)] \quad (10)$$

defines a point in a d -dimensional “embedded” phase space (see, e.g., Farmer & Sidorowich 1987; Casdagli 1989). Given a time T , the idea of a prediction algorithm is to find a mapping f such that $f(\mathbf{X}(T))$ gives a good approximation to $x(T + \Delta t)$. The predictive mapping technique that is used here uses a local approximation method (see, e.g., Casdagli 1989), which considers the behaviour of the nearest neighbours, in phase space, to $\mathbf{X}(T)$. By using a least squares fit, the subsequent evolution of each of these neighbouring points in phase space is used to construct a piecewise-linear approximation to the predictive map, f . This approximate mapping can then be applied to $\mathbf{X}(T)$ to obtain an estimate for $x(T + \Delta t)$. This algorithm can then be repeated to find estimates for $x(T + 2\Delta t)$ and subsequent points. The optimal value for d can be determined by minimising the error of this predictive algorithm over the known segment of the timeseries.

The results of applying this predictor algorithm to the target solution are also shown in Figure 5, where the timeseries predictions are shown as crosses. The prediction is started from the cycle maximum before the mean-field predictor diverges from the target. Longer training timeseries lead to a more densely-populated reconstructed attractor, which increases the probability of making more accurate predictions. However, rather than using the entire target run, these predictions are based upon (approximately) 50 cycles — this will give a fairer comparison between these results and timeseries predictions that are based upon the real sunspot data. The application of the algorithm to earlier segments of the timeseries suggests that a value of $d \geq 5$ is required in order to minimise predictive errors. As can be seen from Figure 5, this algorithm appears to predict the magnitude of the maximum of the following cycle to a reasonable degree of accuracy, although the predictions subsequently diverge from the target. Whilst neither of these techniques are capable of producing reliable long-term predictions, these results do suggest that for the short-term prediction of solar magnetic activity, timeseries analysis techniques may provide a viable alternative to predictions based simply upon mean-field dynamo models (provided stochastic effects can be neglected).

6. Conclusions

Solar magnetic activity arises as a result of a hydromagnetic dynamo — that much we believe to be true. As yet, there is no consensus on the location of the dynamo, the dominant

nonlinear or stochastic effects, or even the fundamental processes that are responsible for the operation of such a dynamo. Although plausible mechanisms have been proposed, as yet none of these are entirely satisfactory. Against this background, there is a drive to be able to predict solar activity with greater accuracy, due to the importance of this activity in driving solar events.

What we have demonstrated here is that no meaningful predictions can be made from illustrative mean-field models, no matter how they are constructed. If the mean-field model is constructed to be a driven linear oscillator then the small stochastic effects that lead to the modulation will have an extremely large effect on the basic cycle and make even short-term prediction extremely difficult. The second scenario, where the modulation arises as a result of nonlinear processes rather than stochastic fluctuations, is clearly a better one for prediction — though here too, prediction is fraught with difficulties. Owing to the inherent nonlinearity of the dynamo system, long-term predictions are impossible (even if the form of the model is completely correctly determined). Furthermore, even short-term prediction from mean-field models is meaningless because of fundamental uncertainties in the form and amplitude of the transport coefficients and nonlinear response. Any deterministic nonlinear model that produces chaotically modulated activity cycles will be faced with the same difficulties.

The equations that describe dynamo action in the solar interior are known to be nonlinear partial differential equations — the momentum equation is nonlinear in both the velocity and the magnetic field. One indication of the role played by nonlinear effects in the solar dynamo is the presence of cyclic variations in the solar differential rotation (the “torsional oscillations”). Furthermore estimates of the field strength at the base of the convection zone consistent with the observed formation of active regions yield fields of sufficient strength ($10^4 - 10^5 \text{G}$) for the nonlinear Lorentz force to be extremely significant, whilst the flows are vigorously nonlinear and turbulent. It therefore seems extremely unlikely that the dynamics of the solar interior can be described by a forced linear system without throwing away much (if not all) of the important physics. In this case it must be argued *not only* that this discarded physics is irrelevant to the dynamo process but also that the parameterisation of the unresolved physics should not include a stochastic component, as this would have an extremely large effect on such a relinearised system.

It is certainly tempting to try to use the observed magnetic flux at the solar surface as an input to a model for prediction (whether nonlinear or stochastic, mean-field or full MHD). Certainly any fully consistent solar activity model constructed in the future should be capable of reproducing the observed pattern of magnetic activity at the solar surface, although this will require a complete understanding not only of the generation process via dynamo action, but also the processes which lead to the formation and subsequent rise of

concentrated magnetic structures from the solar interior to the surface. However it is not clear what role the flux at the solar surface plays in the basic dynamo process. Is it inherent to the process (as modelled by flux transport dynamos) or simply a by-product of the dynamo process that is occurring deep within the sun? Estimates suggest that between 5 and 10% of the solar flux generated in the deep interior makes it to the solar surface (e.g. Galloway & Weiss 1981). For the flux at the solar surface to be the key for dynamo action, it must be explained why the majority of the magnetic flux that resides in the solar interior plays such a little part in the dynamics (to such an extent that it does not even appear as a small stochastic perturbation to the large-scale flux transport dynamo).

Finally it is important to stress that *even if* a model has been tuned so as to reproduce results over a number of solar activity cycles, then there is a good chance of error in the prediction for the next cycle. Any advection-diffusion system in which one is free to specify not only the sources and the sinks but also the transport processes can be tuned to reproduce any required features of activity. Moreover, the formulation of a prediction in terms of a parameterised mean-field model does not inherently put the prediction on a sounder scientific basis than a prediction based on methods of timeseries analysis alone (some of which use very sophisticated mathematical techniques). This, of course, is not to say that any given prediction from such a model will be incorrect, just that the basis for making the prediction has no strong scientific support.

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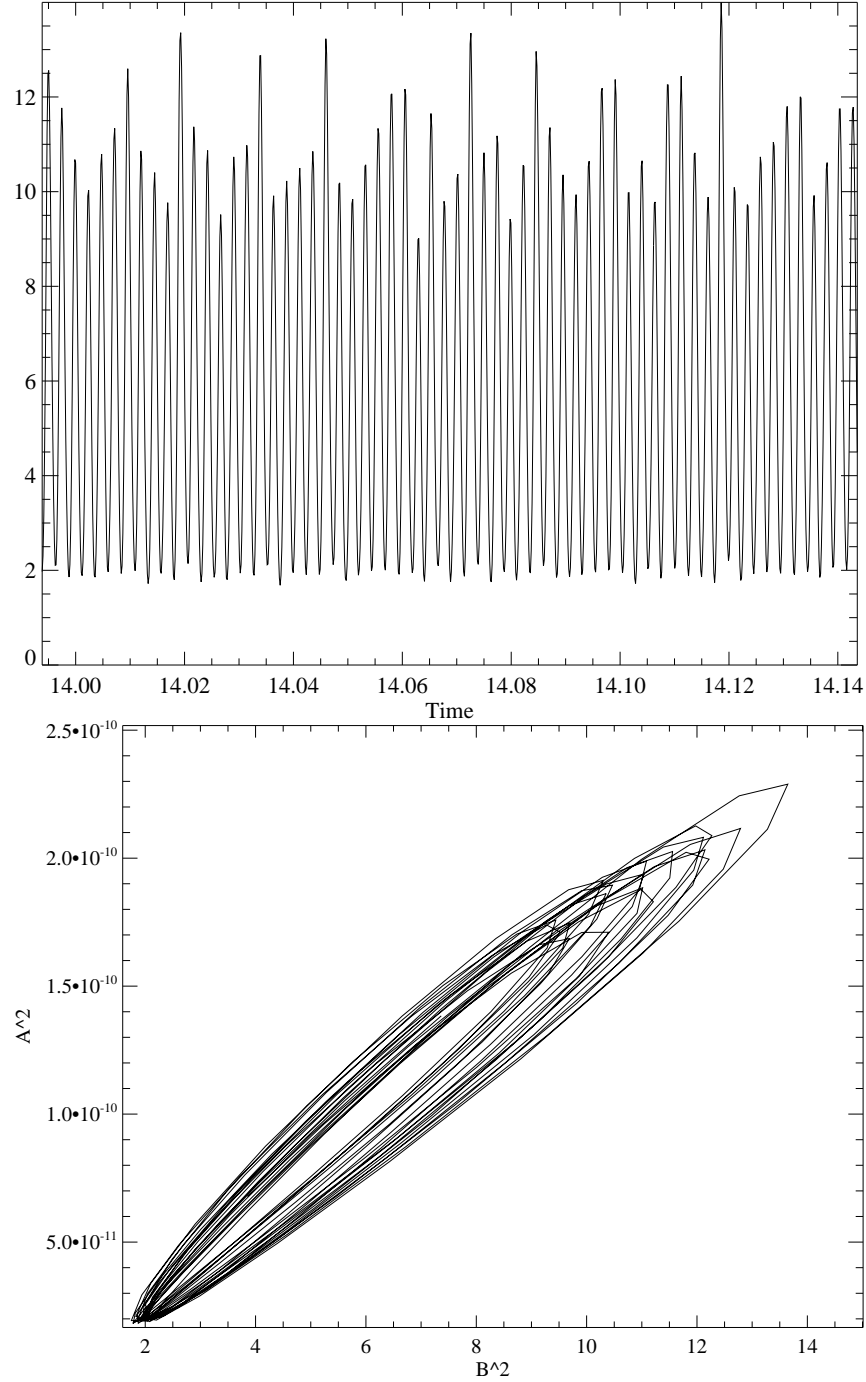


Fig. 1.— The time evolution of the stochastically-perturbed flux transport dynamo. Top: Timeseries for the mean of the squared toroidal field (B^2) at the base of the convection zone. Bottom: In this figure, B^2 at the base of the convection zone is plotted against the mean of the squared values of the poloidal magnetic potential (A^2) at the surface of the domain.

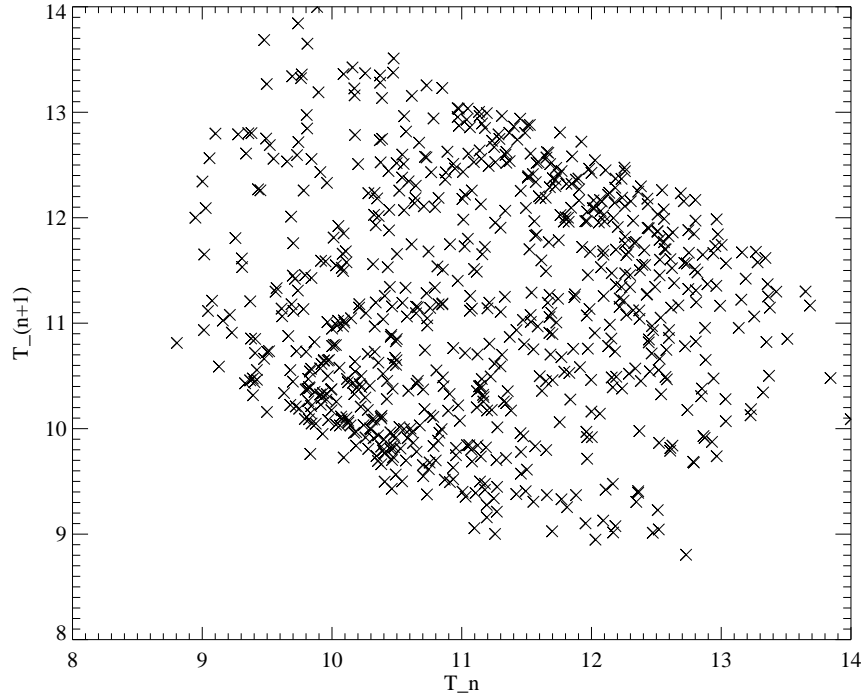


Fig. 2.— The lack of correlation between successive maxima in the stochastically perturbed timeseries (as shown in Figure 1). Defining T_n to be magnitude of the n^{th} maximum, this plot shows the sequential behaviour of these maxima, plotting T_{n+1} as a function of T_n .

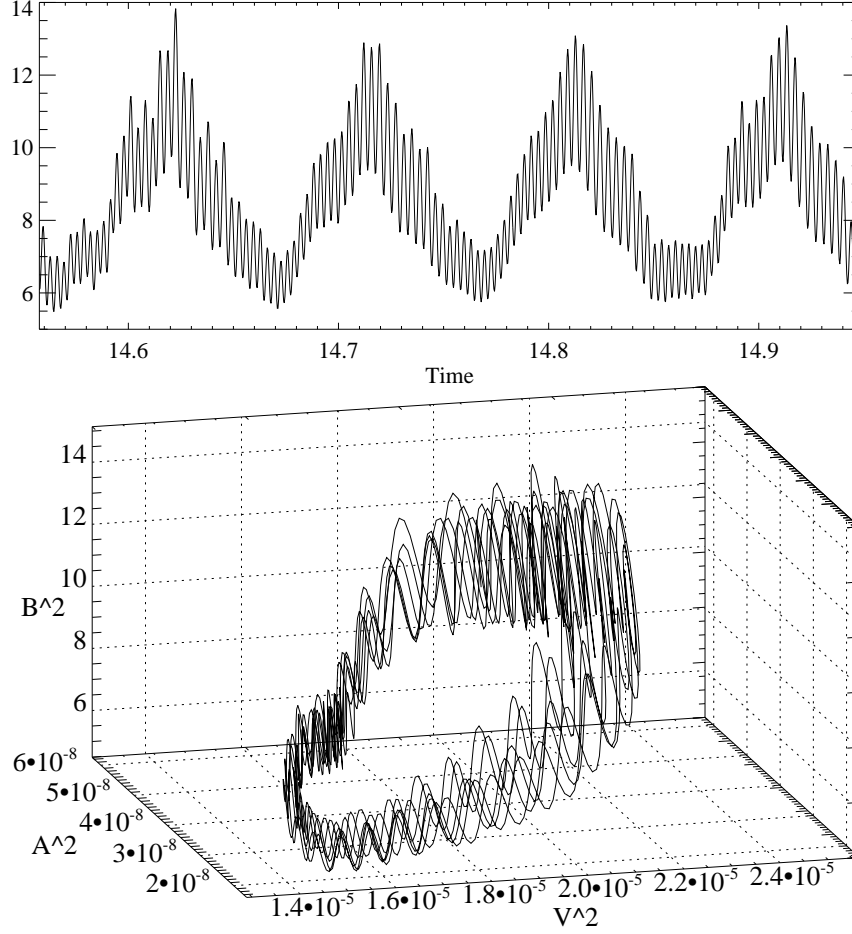


Fig. 3.— The time evolution of the target solution. Top: Timeseries for the mean of the squared toroidal field (B^2) in the dynamo region. Bottom: An attractor for the target solution, in which B^2 is plotted against the mean of the squared values of the poloidal magnetic potential (A^2) and the velocity perturbation (V^2).

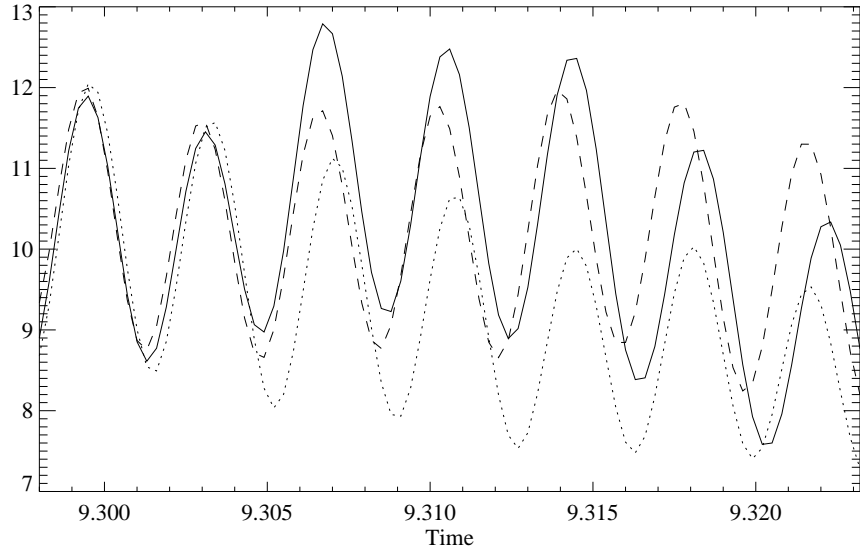


Fig. 4.— Three timeseries showing the evolution of the mean squared toroidal field in the dynamo region. The solid line shows a segment of the target solution timeseries; the dashed and dotted lines show the time-evolution of solutions that are started from nearby points on the same attractor. Although all solutions have the same model parameters, the timeseries rapidly diverge after a couple of cycles.

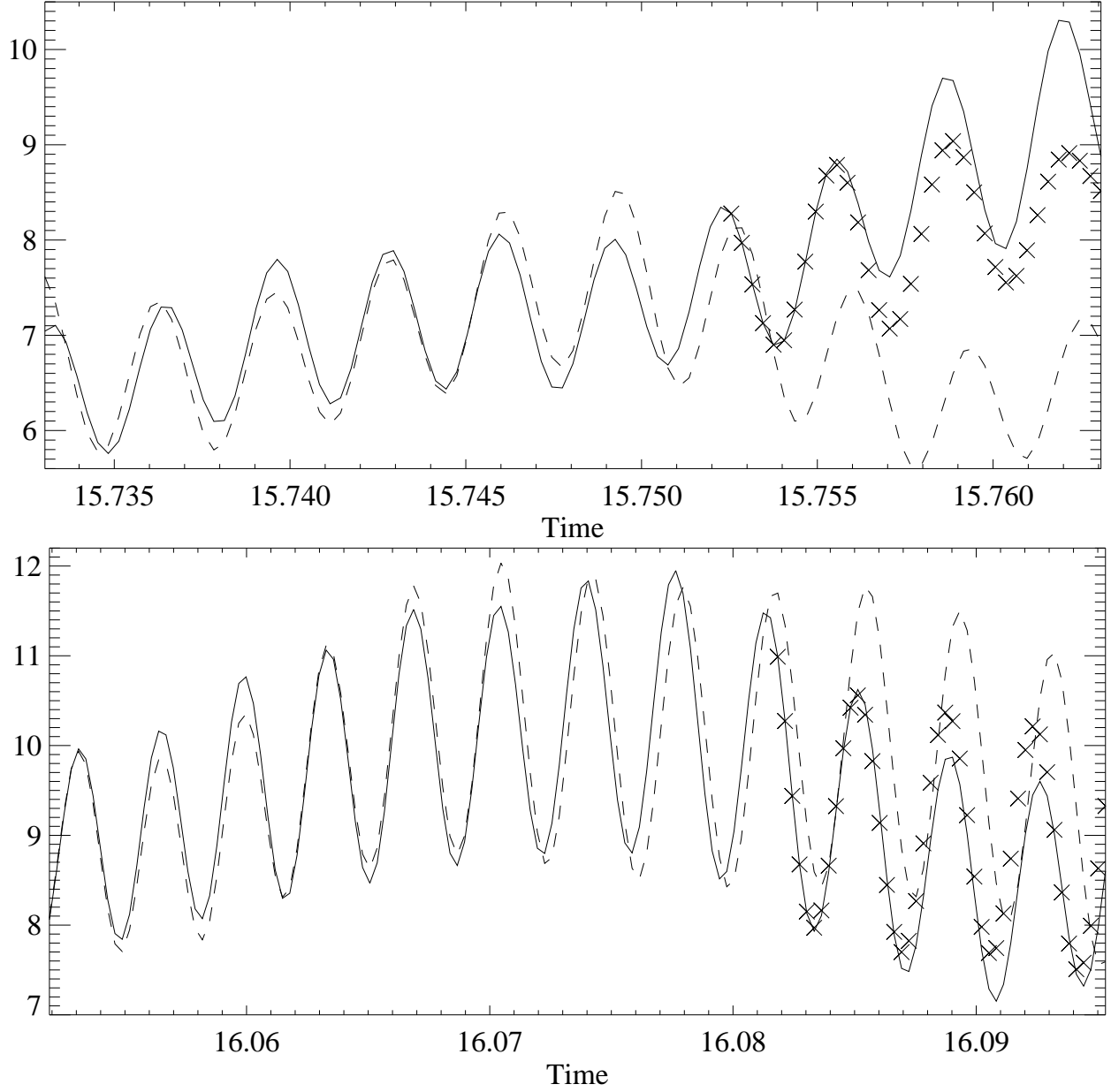


Fig. 5.— Attempts to predict two different segments of the target solution timeseries: In each plot, the solid line shows a segment of the target solution timeseries, the dashed line shows the behaviour of the chosen mean-field predictor (different predictors are used for each plot), whilst the crosses show the predictions that are obtained by reconstructing the nonlinear attractor for the target solution. In each case, the mean-field predictors have been optimised by ensuring that the chosen predictor closely matches the target timeseries segment for a large number of cycles (6 cycles in the upper plot, 9 in the lower).